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Synthesis of optimal multilayer periodic systems: multicriterial approach and realization of synthesized system

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A novel effective approach to formulation and solving of a multilayer system synthesis problem has been developed. The main characteristics of the system spectrum are used as quality criteria to formulate the multicriteria optimization problem. The preliminary analysis of a specific system has been shown to simplify the optimization procedure essentially and to obtain a unique solution of the problem. A set of examples illustrates the efficiency of the developed approach. Physical reasons for deviations of experimentally realized system from the synthesized one have been formulated.

Развит новый подход к постановке и решению задачи синтеза многослойных систем. Основные характеристики спектра системы использованы в качестве критериев качества, на основе которых сформулирована задача многокритериальной оптимизации. Показано, что предварительный анализ конкретной системы позволяет в ряде случаев существенно упростить задачу оптимизации и получить однозначное ее решение. Эффективность предложенного подхода продемонстрирована на ряде примеров. Сформулированы физические причины возможных отклонений характеристики экспериментально реализованной системы от синтезированной.

Multilayer systems (MS) are the most important elements of modern optoelectronic, optical and other devices. This is connected with the fact that multilayer combination of small quantities of materials provides the systems with unique properties which differ fundamentally from those of individual components. Moreover, varying the geometry and the components, it is possible to obtain the systems with preset properties. The MS quality defines to a great extent the characteristics of devices and constructions. In this connection, in modern editions a lot of attention is given to the problem of MS production with preset properties [1–4]. In the framework of MS development for various applications, there are now the following actual tasks: 1) selection and investigation of new materials; 2) development of new synthesis techniques (the synthesis means selection the optimal MS parameters); 3) development of new quality criteria describing adequately the requirements for coating physical characteristics; 4) development of new technologies for MS preparation. These tasks should be solved simultaneously to design most effective new multilayer systems. The tasks 2 and 3 are interrelated, namely, the selection of quality criterion influences the synthesis method and vice versa.

In most cases, the synthesis problem is solved according to the following general scheme. At first, a scalar quality criterion is formulated and formalized to take an extreme value for optimal system (the mathematical model of the system to be synthesized is assumed to be known). Usually, mean square deviation of synthesized system characteristics from ideal ones is used as such criterion. At the second stage, the minimum of the obtained quality criterion is searched for in the space of parameters varied. The problem of such approach application to MS synthesis is connected with a number of mathematical difficulties. The main ones are the following: 1) in general, the number of layers for a system is unknown, so, unknown is the dimension (the number of parameters to be varied) of the task under consideration; 2) mean square criterion includes a lot of similar local minima.

Nowadays, the main systematic way to this problem solution is the needle variation method developed for such a class of problems [5, 6], which uses variational approach to MS synthesis. The main merit of the method is the absence of assumptions on the system structure, i.e. both the layer parameters (optical constants and thicknesses) and their number are selected. Thus, the method is applicable to MS synthesis for any purpose. However, the method has some lacks. In particular, the synthesized coating structure depends essentially on initial approximations [7]. Additionally, the practical realization of the method is followed by substantial difficulties. The method does not guarantee the global character of the minimum found. Thus, today, the problem of MS synthesis in general approach cannot be considered to be solved and requires development and improvement of mathematical methods for its solution. An alternative is the develpoment of special synthesis techniques for some rather narrow class of multilayer systems. Such an approach requires the detailed preliminary analysis of MS of the concrete type for revealing the features which allow to simplify the synthesis problems. In this work, such approach is realized for MS of a specific type.

Among MS, multilayer periodic systems consisting of alternating layers of materials with low (L) and high (H) refraction indices take a particular place. The quarter-wave periodic systems (each layer optical thickness is equal to a quarter of wavelength λ_0) have resonance at a selected wavelength λ_0 [8] and allow producing the mirrors, filters, and other elements for transformation of spectra of various nature which would be optimal for a number of tasks [9]. A usual notation of multilayer periodic system is [A(LH)^{m}A], where A is the environmental medium (air). Thus, refractive indices of the layers L and H as well as multiplicity m of repeating bilayer component are the main parameters of [A(LH)^{m}A] system.

The aim of this work is to analyze such systems in detail, to put a problem of synthesis, and to develop an effective algorithm of its solving taking into account the most of the system features revealed at analysis stage; as well as to illustrate its operation using a real polymer-crystalline system.

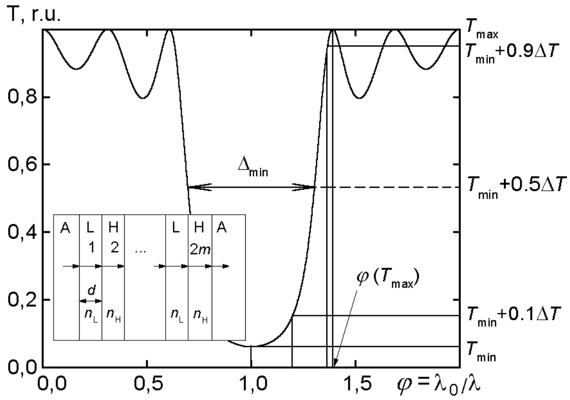

Fig. 1. Typical transmission spectrum and main characteristics of cut-off filter. Inset: The scheme of quarter-wave periodic [A(LH)^{m}A] system.

Multilayer interference systems of [A(LH)^{m}A] type are a basic construction for cut-off filters [10]. In Fig. 1 (inset), a multilayer system with main construction parameters is shown schematically, namely, refractive indices n_L and n_H of low-refractive and high-refractive layers, respectively; multiplicity $m = N/2$ (N is the number of the layers) of elemental two-component system; the layer geometric thickness $d_i = \lambda_0/4n_i$, $i = 1,...2m$; the wavelength λ_0 (main wavelength) where the multilayer system becomes quarter-wave one, i.e. has an extreme due to first order interference. Usually, for convenience of both theoretical analysis and synthesis of multilayer systems, dimensionless value $\varphi = \lambda_0/\lambda$ [1, 5] is used instead of wavelength.

Fig. 1 shows the transmission spectrum of a [A(LH)^{m}A] cutoff system and its main characteristics [10]:

1. Transmission maximum T_{max} at the boundary of operating transparency band;

2. Transmission minimum T_{min} in the high reflection range (background) corresponding to wavelength $\lambda = \lambda_0$ ($\varphi = 1$):

$$T_{\min} = T(\lambda_0) = \left(\frac{2 \cdot \left(\frac{n_H}{n_L}\right)^m}{1 + \left(\frac{n_H}{n_L}\right)^{2m}} \right)^2 \quad (1)$$

3. Cut-off edge sharpness χ is defined by the relation:

$$\chi = \frac{\varphi(T_{\min} + 0.1\Delta T)}{\varphi(T_{\min} + 0.9\Delta T)}, \quad (2)$$

where $\Delta T = T_{max} - T_{min}$, while $\varphi(T_{min} + 0.1\Delta T)$ and $\varphi(T_{min} + 0.9\Delta T)$ are φ values corresponding to transmission on the $(T_{min} + 0.1\Delta T)$ and $(T_{min} + 0.9\Delta T)$ levels;

4. The width of high reflection range Δ_{min} is defined by the value of spectral range $\Delta\varphi$ with transmission at its boundaries $T_{min} + 0.5\Delta T$ (Fig. 1);

5. Contribution of secondary minima into transmission may be described by integral:

$$I_{trans.} = \frac{1}{\varphi(T_{\max})} \int_{\varphi(T_{\max})}^{2} [1 - t(\varphi)]^2 d\varphi. \quad (3)$$

For calculation of MS transmission spectra, the method based on matrix description of thin layer characteristics (Abele method) has been used [8].

Theoretical analysis shows that some values of system parameters may cause not a local minimum at the point $\varphi = 1$ ($\lambda = \lambda_0$) but a local maximum, which is to be get rid of during synthesis of cut-off filters. For this purpose, the second derivative behavior at λ_0 point was analyzed. Positive value of $T_\lambda''(\lambda_0)$ means existance of a minimum at the point λ_0 [11], while negative one, a maximum. The $T_\lambda''(\lambda_0)$ analysis resulted in necessity to limit the range of permissible values for multilayer system parameters by the following conditions:

$$\begin{cases} 1 \le n_L < n_H \le 5, \\ T_\lambda''(\lambda_0, n_H, n_L, m) > 0. \end{cases} \quad (4)$$

The first limitation is naturally imposed on the parameters n_H and n_L, because n_L should be always less than n_H, it takes into consideration as well the physical limitations on the choice of the materials. The second limitation was obtained as a result of $T_\lambda''(\lambda_0)$ analysis. Fig. 2 shows the region of permissible values of 6-layer system parameters according to the condition (4). The further analysis has shown that as the multiplicity m increases, the region corresponding to the condition $T_\lambda''(\lambda_0, n_H, n_L, m) > 0$ becomes wider; hence, the requirements to n_H and n_L parameters become less strict.

For optimal reflecting filter, the transmission at the point λ_0 should be minimal, cutoff sharpness to be maximal, and the

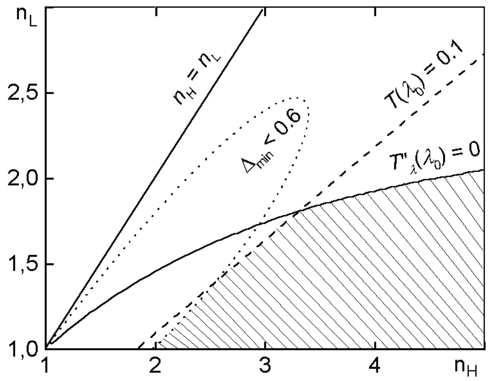

Fig. 2. Diagram n_L and n_H values admitted range for system $[A(LH)^3A]$: the shaded region demonstrates the range of n_L and n_H parameters satisfying the conditions (4)–(6).

number and depths of secondary minima in the operating range (transparency range) to be minimal. At that, the width of the high reflection range should be $\Delta\varphi \ge 0.5$ (see Fig. 1). It is known from literature that the system main characteristics are defined by MS contrast defined by difference between refraction indices of the layers. In some works [5, 10, 12], such difference is formalized using the parameter $\Delta n = n_H - n_L$. However, more detailed analysis carried out in this work has shown that Δn was not a determining parameter for synthesis of optimal periodic systems, but n_L and n_H parameters should be considered independently, especially for small m.

As n_H increases at fixed n_L, the characteristics vary in the following way: $T(\lambda_0)$ tends to zero, while χ, Δ_{min} and I_{trans} increase, and minimum values I_{trans} are attained at the minimum n_H satisfying condition (4). With increasing n_L at fixed n_H, $T(\lambda_0)$ increases (minimum $T(\lambda_0)$ is attained at $n_L = 1$), χ and Δ_{min} decrease (maximum χ values are attained at $n_L = 1$), and I_{trans} grows (minimum values I_{trans} are attained at $n_L = 1$). With increasing m at fixed n_H and n_L, the characteristics vary in the following fashion: $T(\lambda_0)$ trends to zero, χ grows, but the range of high reflection becomes narrower, and both the number and the depth of secondary minima increase. So, a contradiction occurs: m increase results in improvement of some characteristics, while some other ones change for the worse.

The analysis carried out allowed to conclude that to provide optimal MS characteristics, the multiplicity m should not exceed 3 or 4. Moreover, spectral characteristics can be improved more effectively using preliminary selection of the materials with further correction by multiplicity.

Qualitative results of the analysis made are listed in Table 1, where sign <+> means improvement, and <−> means worsening of the spectral characteristics at variation of one of three parameters, two other being fixed. It is obvious that variation of system parameters leads to some characteristic improvement, while others are become worsened. Some characteristics change identically (for instance, $T(\lambda_0)$ and χ). It is worth to note that under increasing the n_L parameter (n_H, m being fixed), all the characteristics change for the worse, i.e. for MS synthesis, the parameter n_L should be chosen as low as possible.

Thus, putting the problem for synthesis of optimal system is complicated by ambiguity of its parameters choice. Moreover, under the synthesis, a simultaneous optimization of several values contradicting to each other is required. Such problem is called the problem of multicriteria optimization and requires the special mathematical methods to be solved.

So, the main spectral characteristics are: transmission minimum in the high reflection range $T(\lambda_0)$, cutoff sharpness χ, high reflection range width Δ_{min}, depths and the number of secondary minima described by I_{trans} integral. They are determined by the main MS parameters: refractive indices of the layers and multiplicity. It is obvious that for ideal filter, each spectral characteristic should take extreme value (maximum or minimum) on above-mentioned MS parameters. That is why these characteristics may be used to formalize the partial criteria of the system optimality [13]. However, application of all the characteristics for multicriteria optimization problem [14] is not obligatory, because some thereof have similar dependences under variation of the system parameters (Table 1). Of χ and $T(\lambda_0)$ characteristics, only one can be taken for partial criterion, for example, χ, while another — $T(\lambda_0)$ — may be represented as a limitation guaranteeing its definite values:

$$T(\lambda_0) \le G_1), \quad (5)$$

where G_1 is a certain positive constant which as a rule has a value of the order of 0.1.

The high reflection range width (Δ_{min}) may be presented as well as a limitation, because its extreme values are not critical; but it is important that it should be rather wide:

$$\Delta_{\min} \ge G_2 \approx \Delta\varphi \approx 0.6, \quad (6)$$

where G_2 is a certain positive constant (Fig. 1).

Table 1. Spectral characteristics of reflecting multilayer periodic system depending on MS parameters

Basic parameters	Characteristics			
	$T(\lambda_0)$	χ	Δ_{min}	$I_{trans.}$
increasing n_H, n_L, m − fixed	+	+	+	−
increasing n_L, n_H, m − fixed	−	−	−	−
increasing m, n_H, n_L − fixed	+	+	−	−

Fig. 2 shows the n_L and n_H values range, satisfying the conditions (4)–(6) for six-layer system [A(LH)^{3}A]. Having fixed the minimum value $n_L = 1$ from the admitted region according to considerations of Section 2, the optimization problem can be reduced to searching for the optimal values of only two parameters: n_H and m. As a result, the multicriteria optimization problem becomes the following form

Criteria:

$$\begin{cases} f_1 = N_1 \cdot (1 - \chi) \to \min, \\ f_2 = N_2 \cdot I_{trans} \to \min, \end{cases}$$

Limitations:

$$\begin{cases} n_L < n_H \le 5, \\ n_L = 1, \\ m = 2,3,,10 \\ T_\lambda''(\lambda_0,n_H,n_L,m) > 0, \\ T(\lambda_0) \le G_1, \\ \Delta_{\min} \ge G_2, \end{cases} \quad (7)$$

where N_i are scale coefficients aimed to reduce all the partial criteria to the [0–1] interval [13].

The generalized criterion is presented as a linear convolution of partial criteria f_i [13]:

$$F(n_H,m) = \alpha f_1(n_H,m) + (1 - \alpha) f_2(n_H,m), \quad (8)$$

where α is the weight coefficient.

The Eq.(8) was minimized as follows: for each multiplicity m, its minimum value was calculated [15] taking into account the limitations (7), among which the minimal one was chosen. This approach is applicable due to discreteness and restrictiveness of a se-

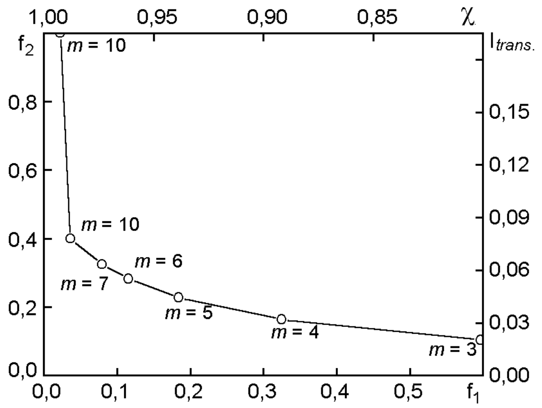

Fig. 3. Pareto set for the problem (7); f_1 criterion characterizes sharpness, and f_2 — secondary extremes.

ries of possible m values and permits reducing the task to 1D minimization problem. This significantly simplifies the calculation process, and is no doubt among the main merits of the approach proposed.

Detailed behavior analysis of each characteristic and, consequently, the respective criterion, depending on the MS parameters has shown that in the region satisfying the conditions (4) to (6) (Fig. 2), their first derivatives are monotonic functions of the system parameters. Consequently, a linear combination of the partial criteria will have not more than a single extreme. Thus, the preliminary analysis has allowed to guarantee the uniqueness of the solution.

Fig. 3 presents the complete solution of the problem for multilayer system synthesis in multi-criteria formulation (7). The line indicates the set of unimprovable solutions (Pareto set) [16] in the space of criteria. The Pareto set is constructed by minimization of generalized quality criterion for different (generally saying, all the possible) values of weight coefficients. It follows from Fig. 3 that minimum multiplicity values are provided by Pareto set points for which sharpness and secondary extremes possess minimal values (f_1 is maximal). The sharpness is the worst, while the secondary extremes are the best. So, the system designer has a possibility to choose the solution corresponding to his concrete task. A drawback of formulation (7) is impossibility to affect the multiplicity value directly. In some cases, this may restrict application of the problem in formulation (7).

As we shall consider in what follows the systems with minimal number of layers, now we propose another mathematical formulation

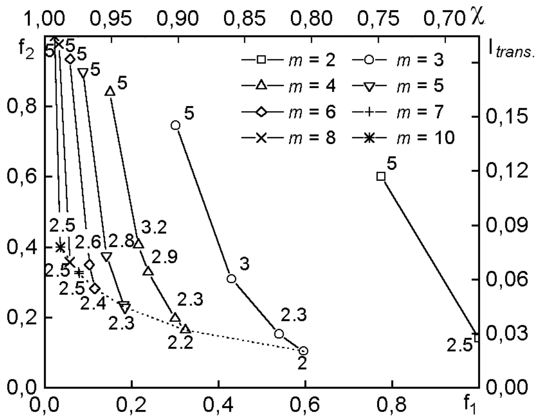

Fig. 4. Pareto set for the problem (9); f_1 criterion characterizes sharpness, and f_2 — secondary extremes; the numbers by the points indicate the n_H values. Dotted line coincides the points corresponding to solution of (7) task.

of synthesis problem where the possibility to affect the system multiplicity is taken into account using additional partial criterion f_3. In this case, the problem (7) takes the form:

Criteria:

$$\begin{cases} f_1 = N_1 \cdot (1 - \chi) \to \min, \\ f_2 = N_2 \cdot I_{trans} \to \min, \\ f_3 = N_3 \cdot m \to \min, \end{cases}$$

Limitations:

$$\begin{cases} n_L < n_H \le 5, \\ n_L = 1, \\ m = 2,3,\ldots,10, \\ T_\lambda''(\lambda_0, n_H, n_L, m) > 0,' \\ T_{\min} \le G_1, \\ \Delta_{\min} \ge G_2, \end{cases} \quad (9)$$

where N_i are scale coefficients.

The generalized criterion is a linear convolution of partial criteria f_i:

$$F(n_H, m) = \sum_{i=1}^{3} \alpha_i f_i(n_H, m), \quad (10)$$

where α_i are weight coefficients.

The complete solution of the multilayer system synthesis problem in multicriteria formulation (9) is given as a family of two-dimensional Pareto sets in the $(f_1 - f_2)$ − space for different multiplicity values m (Fig. 4). Analysis of Fig. 4 shows that addition of one more criterion f_3 results in a variety of solutions for each multiplicity (solid lines in the plot). In the present task, variation of weight coefficients with additional criterion f_3 makes it possible to vary the spectrum parameters at fixed system multiplicity. In particular, at small multiplicities, it is possible to vary sharpness in a rather wide range. Also, it follows from Fig. 4 that selecting the high refraction index material gives the opportunity to decrease substantially the secondary extremes with insignificant sharpness drop at fixed multiplicity. The more is m, the more important the correct choice of the material is, because at $m \geq 6$, one can attain more than twice decrease of the secondary extremes without significant sharpness drop (the lines are almost parallel to $I_{trans.}$).

To demonstrate the efficiency of the technique proposed, the problem of designing the interference filter for far IR range was chosen. For such systems, characteristic is the requirement not only to obtain the desired spectrum but to minimize the system multiplicity m as well [9].

According to the proposed technique, the $[A(LH)^m A]$ type MS was synthesized [17] basing on the material with the lowest refraction index $n_L = 1.5$ (polyethylene) which may be used for a layer with low refraction index. At fixed value $n_L = 1.5$, the optimization problem is reduced to searching for optimal parameters (n_H, m) and takes the following form:

Criteria:

$$\begin{cases} f_1 = N_1 \cdot (1 - \chi) \to \min, \\ f_2 = N_2 \cdot I_{trans.} \to \min, \\ f_3 = N_3 \cdot m \to \min, \end{cases}$$

Limitations:

$$\begin{cases} n_L < n_H \leq 5, \\ n_L = 1.5, \\ m = 2,3,\ldots,10, \\ T_\lambda''(\lambda_0, n_H, n_L, m) > 0, \\ T(\lambda_0) \leq G_1, \\ \Delta_{\min} \geq G_2. \end{cases} \quad (11)$$

Fig. 5 shows the complete solution of the synthesis problem for such multilayer system in multicriteria formulation (11) as a family of two-dimensional Pareto sets in ($f_1 - f_2$) space for different multiplicities m.

The materials which may be used in MS for far IR as the high refraction index layers combined with polyethylene are KRS-5 ($n = 2.2$), LiF ($n = 3$), and Ge($n = 4$) [9]. From Fig. 5, it is seen that at the points A ($n_H = 3.1$) and B ($n_H = 4.2$), the values n_H are most close numerically to n values of the materials mentioned (LiF and Ge), while there are none of points with n_H numerically close to the value $n = 2.2$ (KRS-5).

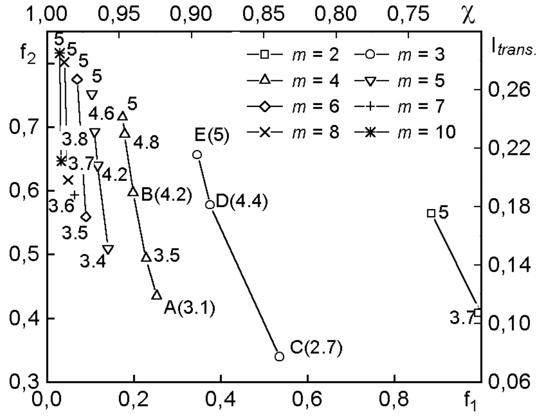

Fig. 5. Pareto set for the problem (11); f_1 criterion characterizes sharpness, and f_2 — secondary extremes; the numbers by the points indicate the n_H values.

The characteristics of the system corresponding to point A are: $T(\lambda_0) = 0.01$, $\chi = 0.9$, $\Delta_{min} = 0.6$, $I_{trans.} = 0.1$; to point B: $T(\lambda_0) = 0.001$, $\chi = 0.93$, $\Delta_{min} = 0.75$, $I_{trans.} = 0.18$. The system corresponding to B point has just a bit different sharpness in comparison with A point, but deeper secondary extremes. Thus, of the systems under consideration, the one corresponding to A point ($n_H = 3.1$) is most preferable. However, from the technological considerations, for far IR it is important to lower the multiplicity of the system synthesized [9, 17].

In the Pareto set, three points correspond to multiplicity $m = 3$ (Fig. 5): C ($n_H = 2.7$, $T(\lambda_0) = 0.1$, $\chi = 0.82$, $\Delta_{min} = 0.61$, $I_{trans.} = 0.07$), D ($n_H = 4.4$, $T(\lambda_0) = 0.006$, $\chi = 0.88$, $\Delta_{min} = 0.84$, $I_{trans.} = 0.18$), and E ($n_H = 5$, $T(\lambda_0) = 0.002$, $\chi = 0.89$, $\Delta_{min} = 0.9$, $I_{trans.} = 0.21$). It is obvious that the C point has n_H numerically closer to n of LiF, and the point D, to that of Ge. The MS corresponding to D point has better spectrum characteristics $T(\lambda_0)$ and χ, but deeper secondary extremes. Of the systems considered, most preferable is the one which has better sharpness and lower background, i.e. the system corresponding to D point. So, in Pareto set, the point has been chosen theoretically which corresponds to the system with minimal multiplicity ($m = 3$), low background ($T(\lambda_0) = 0.006$), good sharpness

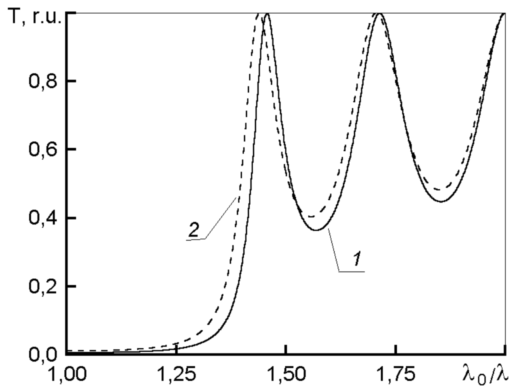

Fig. 6. Transmission spectra of six-layer $[A(HL)^3A]$ system with parameters $n_L = 1.5$; $n_H = 4.4$ (*1*) and $n_H = 4$ (*2*).

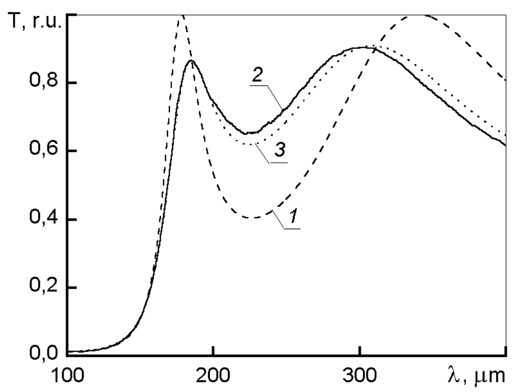

Fig. 7. Transmission spectra of six-layer system with parameters $n_L = 1.5$; $n_H = 4$: *1* — synthesized system; *2* — experimentally realized system; *3* — system synthesized taking into account both absorption in L-layers ($k = 0.01$) and random layer thickness deviations.

($\chi = 0.88$) and wide high reflection range ($\Delta_{min} = 0.84$). For the point chosen $n_H = 4.4$, that is closest to n value of Ge ($n = 4$).

In Fig. 6, theoretically calculated transmission spectra of $[A(LH)^3A]$ MS with parameters $n_L = 1.5$, $n_H = 4.4$ and $n_H = 4$ are shown; it is seen that the spectra are essentially the same.

Fig. 7 shows transmission spectra of synthesized (curve *1*) and experimentally realized (curve *2*) multilayer $[A(HL)^3A]$ systems with parameters $m = 3$, $n_L = 1.5$, and $n_H = 4$. It is seen that the produced MS exhibits some quantitative differences as compared to calculated one. These differences may be related to idealization of the model used for synthesis. In this model, the following factors were not taken into consideration: 1) dispersion of refractive indices of the layer materials; 2) absorption in the layers; 3) thickness errors of deposited layers (deviations from quarter-wave rule); 4) scattering due to roughness of the layer interfaces and micro-inhomogeneities in the layer volumes. Curve *3* in Fig. 7 shows that taking into account the factors 2 and 3 provides substantially similar spectra of experimental and synthesized systems.

To conclude, a new technique for synthesis MS based on multicriteria optimization is proposed. Detailed analysis of MS spectrum characteristics as functions of the main parameters is the physical basis for partial criteria choice. The analysis has allowed: 1) to show that no MS parameter selection provides the simultaneous improvement of all the characteristics; 2) to construct the admitted region for the system parameters values; 3) to reduce the number of the fitting parameters; 4) to formulate partial quality criteria; 5) to formalize the synthesis problem in multicriteria approach; 6) to simplify optimization procedure and to obtain a single-valued solution of the problem. Using the synthesis problem solution for polyethylene-germanium quarter-wave MS as an example, it was shown how the approach developed may be adopted to synthesis of systems with specific requirements. The polyethylene-germanium system has been synthesized. Physical causes for deviations of experimental characteristics from calculated ones have been considered.

References

1. K.D.Misra, R.K.Mishra, *Optika i Spectr.*, **97**, 475 (2004).
2. D.G.Makarov, V.V.Danilov, V.F.Kovalenko, *Optika i Spectr*, **97**, 632 (2004).
3. S.A.Fetisenkov, K.N.Krivetsky, *Opt.J.*, **71**, 88 (2004).
4. A.G.Gorshkov, E.I.Starovoitova, A.V.Yarovaya, Mechanics of Layered Viscous-Elasic-Plastic Construction Elements, Fizmatlit, Moscow (2005) [in Russian].
5. Sh.Furman, A.V.Tikhonravov, Basics of Optics of Multilayer Systems, Editions Frontiers, Gif-sur-Yvette (1992).
6. A.G.Sveshnikov, A.V.Tikhonravov, M.N.Trubetskov, *Mathem. Model.*, **7**, 105 (1995).
7. P.Baumeister, *Appl. Optics*, **34**, 4835 (1995).
8. M.Born, E.Wolf, Principles of Optics, Pergamon, New York (1999).
9. A.I.Belyaeva, S.N.Kolomiets, *Functional Materials*, **11**, 620 (2004).
10. A.I.Belyaeva, V.I.Sirenko, Cryogenic Multilayer Coatings, Naukova Dumka, Kyiv (1991) [in Russian].
11. G.M.Fikhtengolts, The Principles of Mathematical Analysis, Lan', v.1, Moscow (2002) [in Russian].

12. V.F.Suyetin, M.N.Cherepanova, A.F.Perveyev, *Opt. Mekhan. Prom.*, No.6, 15 (1963).
13. R.Schtoyer, Multi-criteria Optimization: Theory, Calculations and Appendices, Radio i Svyaz', Moscow (1992) [in Russian].
14. R.L.Kini, H.Rayfa, Taking the Decisions under Many Criteria: Preferences and Substitutions, Radio i Svyaz', Moscow (1981) [in Russian].
15. A.F.Izmailov, M.V.Solodov, Numerical Methods of Optimizaton, Fizmatlit, Moscow (2005) [in Russian].
16. V.D.Nogin, Taking the Decisions in Multi-criteria Medium: Quantitative Approach, Fizmatlit, Moscow (2002) [in Russian].
17. A.I.Belyaeva, S.N.Kolomiyets, *Fiz. Tekhn. Vysok. Davl.*, No.14, 96 (2004).

Синтез оптимальних багатошарових періодичних систем: багатокритеріальний підхід та реалізація системи, що синтезована

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Розвинуто новий підхід до постановки і рішення задачі синтезу багатошарових систем. Основні характеристики спектра системи використовуються як критерії якості, на основі яких сформульовано задачу багатокритеріальної оптимізації. Показано, що попередній аналіз конкретної системи дозволяє у ряді випадків істотно спростити задачу оптимізації і одержати однозначне її рішення. Ефективність запропонованого підходу продемонстровано на ряді прикладів. Сформульовано фізичні причини можливих відхилень характеристики експериментально реалізованої системи від синтезованої.