

## The slow electromagnetic wave effect induced by the interaction of dark and quasi-dark modes in microwave metamaterials

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The work is devoted to the development and numerical simulation of a planar microwave metamaterial demonstrating the effect of two-mode plasmon-induced transparency (PIT effect) in the transmission spectra when a plane electromagnetic wave is incident on the surface of the metamaterial at the frequency of the fundamental surface lattice mode. The resonance responses of the metamaterial and the action of slow electromagnetic waves in the microwave frequency range of 50–60 GHz are investigated. It has been shown by the method of numerical simulation that windows of two-mode transparency in the fundamental surface lattice modes can appear for a given topology and geometric parameters of the metamaterial structure. The correlation between the structural parameters of the unit cell of the metamaterial, the spectral transmission coefficients and the group delay time of the incident electromagnetic waves has been determined. The developed computer model of a planar metamaterial demonstrated large values of the group refractive index, the delay time of the group velocity of electromagnetic excitation and the product of the bandwidth and the delay time.

**Keywords:** metamaterial, unit cell, plasmon induced transparency effect, "bright" modes, "dark" modes, surface lattice resonance modes, transparency windows, DBP.

**Ефект повільної електромагнітної хвилі, викликаний взаємодією темної та квазітемної мод у мікрохвильових метаматеріалах.** Ю.Н.Савін, З.Є.Єременко, О.А.Бреславець

Проведене дослідження присвячено розробці і чисельному моделюванню планарних мікрохвильових метаматеріалів, які демонструють ефект двомодової плазмон-індукованої прозорості (PIT-ефект) у спектрах пропускання під впливом плоскої електромагнітної хвилі на поверхні метаматеріалу на частоті фундаментальної поверхневої моди решітки. Вивчено резонансні відгуки та ефект уповільнення електромагнітних хвиль у мікрохвильовому частотному діапазоні 50–60 ГГц. Методом чисельного моделювання продемонстровано, що двомодові вікна прозорості у спектрі фундаментальної поверхневої решіткової моди можуть з'являтися при певній топології та геометричних параметрах елементарної комірки метаматеріалу. Визначено кореляцію між структурними параметрами елементарної комірки, спектральними коефіцієнтами пропускання та часом затримки групової швидкості електромагнітних хвиль, що падають на поверхню метаматеріалу. Чисельна модель планарного метаматеріалу, яку розроблено, демонструє великі значення групового показника рефракції, часу затримки групової швидкості електромагнітного збудження та добутку групової затримки на пропускну здатність.

Работа посвящена разработке и численному моделированию планарного микроволнового метаматериала, демонстрирующего эффект двухмодовой плазмон-индуцированной прозрачности (PIT-эффект) в спектрах пропускания при падении плоской электромагнитной волны на поверхность метаматериала на частоте фундаментальной поверхностной решеточной моды. Изучены резонансный отклик и эффект замедления электромагнит-

ных волн в метаматериале в микроволновом спектральном диапазоне 50–60 ГГц. Методом численного моделирования показано, что двухмодовые окна прозрачности в спектре фундаментальной решеточной моды могут наблюдаться при определенной топологии и геометрических параметрах элементарной ячейки метаматериала. Определена корреляция между структурными параметрами элементарной ячейки метаматериала, спектральными коэффициентами пропускания и временем групповой задержки падающих на поверхность метаматериала электромагнитных волн. Разработанная компьютерная модель планарного метаматериала продемонстрировала большие значения группового показателя преломления, времени задержки групповой скорости электромагнитного возбуждения и произведения ширины полосы на время задержки.

### 1. Introduction

The slow light effect is one of the characteristic features of electromagnetically induced transparency (EIT) resonance. The EIT-effect is a quantum-mechanical phenomenon observed in atomic or molecular systems, which results in a spectrally narrow optical transparency band accompanied by extreme frequency dispersion [1, 2]. This property is a key to achieving a significant reduction in the group velocity of propagating light and can be used in a variety of optical devices, including ultrafast switches, modulators, advanced nonlinear optical systems, slow light, and optical storage devices [3–5].

At present, much attention is paid to various structures of electromagnetic metamaterials with a resonant response that mimics the spectral response of EIT (EIT metamaterials). Such EIT-metamaterials usually consist of so-called subwavelength metamolecules [6] structured in a periodic array of metal resonators on the surface of a dielectric substrate. Due to their subwavelength periodicity, they interact homogeneously with the incident electromagnetic field and can exhibit strong resonant responses and the plasmon induced transparency effect (PIT-effect). The physical nature of this effect is based on the near-field coupling and destructive interference of the so-called superradiant "bright modes" and subradiant "dark mode" of different types of metallic meta-atoms localized in the unit cell [7]. The "bright" mode is due to the strong interaction of the incident electric field with the collective oscillations of conductive electrons in a metallic microresonator and exhibits a wider resonance and a lower quality factor. On the contrary, the "dark" mode either does not interact or interacts very weakly with the incident electric field and leads to a sharper resonance with a higher  $Q$  factor than for the bright mode. "Quasi dark" modes can interact with the incident electric field. However, as compared to the "bright" mode, this interaction is several times weaker and the quality factor of resonance response is much higher [8–10].

Over the past period, a large variety of PIT metamaterials have been proposed and demonstrated at microwave [11], terahertz [12], infrared and optical regimes [13]. Since metamaterials are periodic structures, there is another type of dark modes, the so-called "quasi-dark" modes, which are surface lattice modes (SLR modes). In general, the full bandwidth at half maximum (FWHM) of the SLR mode is much smaller than the FWHM in plasmonic "bright" meta-atoms, but can be much larger than in specially constructed "dark" meta-atoms [14].

Due to the trapped fields, the lattice mode behaves like a quasi-dark mode with respect to the "bright" mode of the meta-atom and its resonant frequency can be easily controlled by changing the periodicity of the array (that is, the period of the unit cell). The interaction of the lattice mode with "bright" meta-atoms leads to the lattice induced transparency (LIT-effect) of the metamaterial with a single window transparency. This phenomenon is well studied [15–17]. At the same time, the interaction between the quasi-dark lattice mode and the dark mode of the polyatomic structures (metamolecules), which can cause the PIT effect on high-quality resonance modes of the lattice, is much less studied. The PIT-effect on lattice modes can cause a thin structure of the transmission/reflection spectra, a higher  $Q$  factor and sharp frequency dispersion in the low frequency range, and, therefore, a large group delay in time and a slow electromagnetic wave transmitted through metamaterials. Changing the resonance intensity, FWHM and frequency in the PIT window can be performed by controlling the lattice period and the geometric size of metamolecules in the metamaterial unit cell.

The aim of this work is to design, numerically simulate, and determine the functional response of microwave metamaterials based on an ordered array of plasmonic meta-atoms and the PIT effect, demonstrating large values of slow electromagnetic waves and group refractive indices.

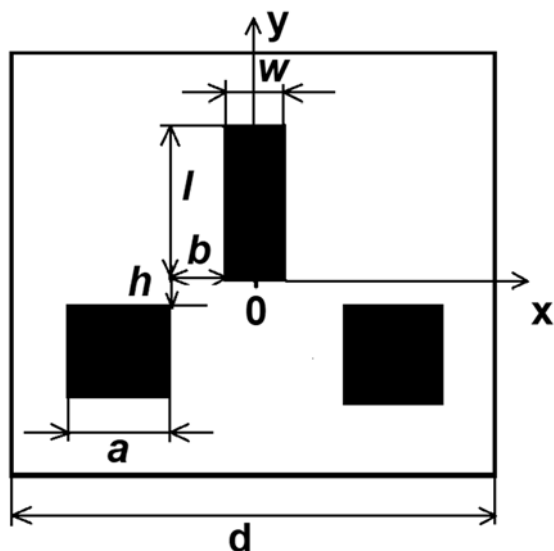


Fig. 1. The structure of a unit cell with three meta-atoms:  $a$  is the square side (variable);  $b$  and  $h$  are distances between "bright" and "dark" meta-atoms;  $l$  and  $w$  are the sizes of a "bright" atom;  $d$  is the unit cell period.

## 2. Methodology

The main method is numerical simulation of metamaterial structures and their resonance responses. For the numerical simulation of metamaterials we used the Frequency Domain Solver module in the CST Microwave Studio software product (Student Edition) [18]. This solver made it possible to calculate electromagnetic fields near the metamaterial surface by the Finite Element Method. In particular, this software module was used to numerically simulate the spectra of transitions and profiles of the electromagnetic field in the near band. The Frequency Domain Solver module was used to calculate the  $S_{12}$ -parameters of the scattering matrix and also to simulate the intensity distributions of electromagnetic fields in unit cells of the metamaterial under study. The transmission spectra were obtained under the following conditions: the unit cell was excited by  $TE$  polarized plane electromagnetic field normally to the metamaterial surface and the numerical solution of Maxwell's equations was carried out under periodic boundary conditions.

## 3. Results and discussion

The constructed and studied metamaterial unit cell shown in Fig. 1 consist of three metal cut wire resonators ( $CWR_i$ ), where  $i = 0, 1, 2$  corresponds to meta-atoms with different shapes and geometrical sizes. The meta-atom  $CWR_0$  is a micro-strip  $l =$

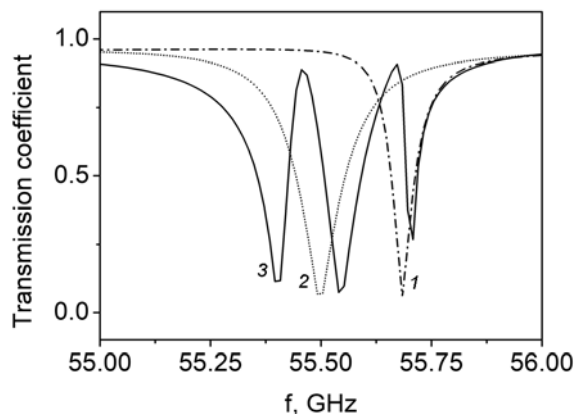


Fig. 2. Transmission spectra of the fundamental SLR mode of a meta-molecule with a single "bright" atom ( $CWR_0$ ) (1); with a pair of "dark" atoms (2); spectra of triatomic meta-molecules (3).  $b = 0.7$  mm,  $h = 0.1$  mm,  $a = 1.12$  mm,  $l = 1.6$  mm,  $w = 0.7$  mm.

1.6 mm long and  $w = 0.7$  mm wide. The  $CWR_1$  and  $CWR_2$  are square-shaped. These meta-atoms are placed symmetrically with respect to  $CWR_0$  at the distance  $b = 0.7$  mm. Both the unit cell period and the side size of  $CWR_1$  and  $CWR_2$  are variable parameters. The  $CWR_i$  materials are: copper (electrical conductivity is  $5.96 \cdot 10^7$  S/m, thickness is 0.035 mm), the substrate material is Teflon (dielectric constant 2.1, dielectric losses is 0.0002, substrate thickness is 1.2 mm). Unit cells are excited by a linear polarized electromagnetic field in the microwave range 40–60 GHz in  $TE$ -polarization perpendicular to the surface of the metamaterial.

The results of the simulation have shown that the unit cell with a single  $CWR_0$  has two spectral transmission modes: a "bright" mode band at 48.56 GHz with a FWHM bandwidth 0.4 GHz and the quality factor  $Q = 108$  (this band is due to the interaction of surface plasmons in  $CWR_0$  with inner electromagnetic field) and the "quasi-dark" mode band at 55.68 GHz, with a narrow FWHM bandwidth 0.057 GHz and the high quality factor  $Q = 1114$  (Fig. 2). This mode is a first-order surface lattice resonance (SLR) mode which arises due to enhanced radiative interaction between the localized resonances in the  $CWR_i$  scatterers through diffraction orders in plane of the unit cell [19].

As is known, surface lattice resonance modes are mediated by surface plasmon-polaritons excited in a periodic array of metal

microresonators, when an electromagnetic wave is incident on a diffractive surface. At normal incidence of an EM wave on a square array with period  $d$ , the lattice modes excited at different frequencies obey the following relationship [17]

$$f_{Lkm} = \left( \frac{c}{n_s d} \right) \sqrt{k^2 + m^2}, \quad (1)$$

where  $f_{Lkm}$  is the frequency of the diffracted lattice mode;  $n_s$  is the refractive index of the substrate; and  $c$  is the speed of light in vacuum. A pair of integers  $(k, m)$  determines the order of the diffraction mode. We only match the first order diffraction mode (1.0) or (0.1), since this is the mode that has the maximum intensity of the trapped electromagnetic wave in the meta-surface plane. The numerical simulation of spectral responses has demonstrated the frequency dependences of the plasmonic dipole mode and the lattice mode on the parameters  $d$  and  $l$ .

The spectra of transitions of a unit cell with a pair of meta-atoms  $CWR_1$  and  $CWR_2$  have only one narrow spectral band close to the Rayleigh cutoff wavelengths [12]. In particular, when the lattice period is  $d = 5$  mm and the side size of  $CWR_1$  and  $CWR_2$  is  $a = 1.1$  mm, the transmission band appears at 55.69 GHz with  $Q = 977$  corresponding to the first SLR mode (Fig. 2). At the same time, when the unit cell consists of  $CWR_1$  (or  $CWR_2$ ), two resonance modes appear in the transmission spectrum. In particular, the transmission bands appear at the eigenvalue of the frequency of 50.71 GHz with a quality factor  $Q = 460$ , as well as at 55.93 GHz with  $Q = 470$ . The first is the plasmonic mode of  $CWR_1$  and the second is the SLR mode. The comparable values of their quality factors mean that the  $CWR_1$  (or  $CWR_2$ ) in the unit cell are not "dark"-mode meta-atoms, but the pairs  $CWR_1$  and  $CWR_2$  with the same geometrical dimensions of meta-atoms, localized symmetrically with respect to the direction of polarization of the internal electromagnetic wave, act as a diatom "dark" meta-molecule. The simulation of transmission spectra of a unit cell with the  $CWR_1$  and  $CWR_2$  pairs has shown the dependences of frequency localization and FWHM bandwidth of the SLR mode on the parameters  $d$  and  $a$ .

The simulation of resonance response of the unit cell for a triatomic meta-molecule with meta-atoms  $CWR_0$ ,  $CWR_1$  and  $CWR_2$  has shown that the triple structure trans-

mission spectra with two transparency windows can exist at some correlated values of the lattice parameter  $d$  and side  $a$ . This triple structure of the lattice spectral band indicates that the PIT effect is induced in the first order SLR mode due to the near-field interaction between (0.1) the "quasi-dark" SLR mode and the "dark" plasmonic mode of the  $CWR_1$  and  $CWR_2$  meta-atom pair. As is known, the necessary conditions for the appearance of the PIT effect in the transmission spectra of plasmonic metamaterials are the overlap of the frequencies of the interacting modes, as well as a large difference in its  $Q$ -factors. In this case, the energy transport can occur from the "bright" mode to the "quasi-dark" or "dark" modes [7]; in our case, the energy transport occurs from the "quasi-dark" mode (i.e. SLR mode) to the "dark" mode (i.e.  $CWR_1$  and  $CWR_2$  pair mode).

To clarify the character of the double PIT effect, we have studied the distribution of electric fields over the unit cell surface at every PIT window. We have found that the excitation of lattice modes in a triatomic unit cell at frequencies  $f_1 \leq 55.423$  GHz,  $f_3 = 55.567$  GHz and  $f_5 \geq 55.711$  GHz leads to the low electric field strength near the  $CWR_1$  and  $CWR_2$  atoms. At the same time, the excitation of a three-atom meta-molecule at frequencies  $f_2 = 55.482$  GHz and  $f_4 = 55.683$  GHz results in the concentration of the electric field at  $CWR_1$  and  $CWR_2$  meta-atoms. As shown earlier, a diatomic meta-molecule (that is a meta-molecule composed of  $CWR_1$  and  $CWR_2$  atoms) cannot be directly excited. Therefore, the excitation of the "dark" modes in the  $CWR_1$  and  $CWR_2$  atomic pair can occur indirectly due to energy transfer from the SLR mode to  $CWR_1$  and  $CWR_2$  atoms. As a result, the interference between electromagnetic fields of the "dark" (from the  $CWR_1$  and  $CWR_2$  pair) and "quasi-dark" SLR modes sharply reduces the radiative losses, and two transparency windows arise in the SLR bandwidth of the transmission spectrum. It can be assumed that the appearance of two transparency windows is associated with doubly degenerate dark modes in a triatomic meta-molecule. This degeneracy is removed, when the meta-molecule interacts with the electric field near SLR and transparency windows appear.

We study the effect of the meta-molecule and unit cell geometrical dimensions on the spectral response of metamaterials. As a result of the numerical simulation it was

Table 1. Correlation of unit cell geometrical parameters of a meta-material unit cell with slow electromagnetic wave parameters for low- and high-frequency transparency windows in the transmission spectrum of the basic SLR mode:  $d$  is the unit cell dimension;  $a$  is the side size of CWRi meta-atoms;  $t_{gr1}$ ,  $t_{gr2}$  are group time delays for two transparency windows — low-frequency (index 1) and high-frequency (index 2);  $n_{gr1}$ ,  $n_{gr2}$  are group refractive indexes;  $f_{gr1}$ ,  $f_{gr2}$  are frequencies of  $t_{gr1}$  and  $t_{gr2}$  peaks respectively,  $T_{gr1}$ ,  $T_{gr2}$  are the transparency peaks in corresponding transparency windows.

$d$ , mm	$a$ , mm	$t_{gr1}$ , ns	$n_{gr1}$	$f_{gr1}$ , GHz	$T_{gr1}$ , %	$t_{gr2}$ , ns	$n_{gr2}$	$f_{gr2}$ , GHz	$T_{gr2}$ , %
4.88	0.94	1.17	290	56.82	93	17.41	4352	56.9	63
4.95	1.05	3.3	825	56.03	90	9.85	2462	56.17	85
5.0	1.12	4.47	1118	55.46	89	6.51	1628	55.67	90
5.1	1.23	5.09	1271	54.31	88	5.65	1413	54.69	93
5.2	1.35	6.37	1567	53.26	85	5.11	1278	53.78	94
5.25	1.4	6.52	1632	52.78	83	4.89	1223	53.34	94
5.35	1.45	7.46	1873	52.54	82	4.87	1218	53.12	94
5.40	1.5	6.88	1588	51.36	80	3.81	977	52.09	92

found that the transmission spectral band with two transparency windows (conventionally, high- and low-frequency) can appear at a certain correlated  $d$  and  $a$  values (Table). With an increase in the parameters  $d$  and  $a$ , both the low-frequency and high-frequency transparency windows in the SLR bandwidth displace to the red transmission spectral range. When  $d < 4.88$  mm and  $a < 0.9$ , the triplet spectral structure degenerates into a doublet. The largest values of  $d$  and  $a$ , for which two transparency windows still exist, are limited by the maximum possible value of CWR<sub>1</sub> and CWR<sub>2</sub> in the unit cell.

We have studied the dependence of the distance ( $\Delta f_L = f_{gr2} - f_{gr1}$ ) between maximum transparency values in the low frequency ( $f_{gr1}$ ) and high frequency ( $f_{gr2}$ ) windows *versus* the distance between CWR<sub>1</sub> and CWR<sub>2</sub> meta-atoms (parameter  $b$ ) in the unit cell. We have previously assumed that the transparency windows appear due to the transfer of SLR mode energy to the CWR<sub>1</sub> and CWR<sub>2</sub> meta-atom pair and the excitation of a doubly degenerated dark mode. At the same time, the removal of the degeneracy of this mode can occur due to dipole interaction between excited CWR<sub>1</sub> and CWR<sub>2</sub> atoms. But as the numerical simulation has shown, when the distance between CWR<sub>1</sub> and CWR<sub>2</sub> atoms increases,  $\Delta f_L$  does not rise but decreases nonlinearly. This means that CWR<sub>1</sub> and CWR<sub>2</sub> atoms don't interact with each other directly and the degeneracy is removed probably due to interaction with SLR modes.

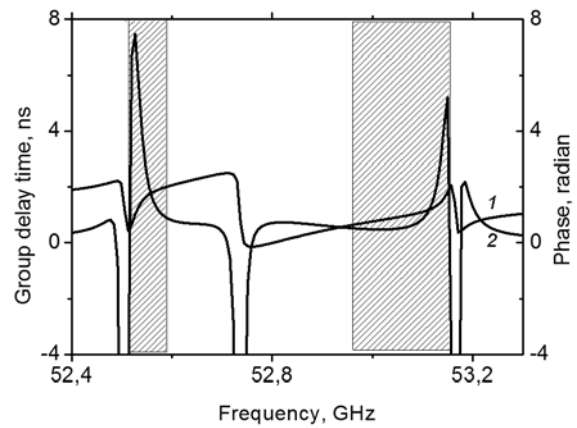


Fig. 3. Change in the phase of an electromagnetic wave passing through the meta-surface (1) and the group delay time (2) depending on the frequency for the unit cell metasurface  $d = 5.35$  mm.

As is known, the most important characteristic of the PIT-effect in the metamaterial is slow propagation of an electromagnetic wave, which is characterized by the group delay time  $t_{gr}$  of the electromagnetic wave passing through the metamaterial medium. Due to the near-field interaction between the fundamental SLR mode of the surface lattice and the localized plasmon mode in the unit cell, there can be a large change in the transmittance in the narrow frequency region of the SLR mode; this causes a stronger phase dispersion that occurs near the transparency window of the SLR mode. As a result, larger values of the group time delay and the group refractive index can occur.

We calculate the group delay value  $t_{gr}$  at various  $d$  parameters for two transparency windows in the basic (0,1) surface lattice mode in according with

$$t_g = \frac{d\phi}{2\pi df}, \quad (2)$$

where  $\phi$  is the transmission phase shift of the electromagnetic wave [20].

Fig. 3 shows corresponding phase shifts and group delay times for a triatomic meta-molecule in the unit cell for different  $d$  values.

The numerical simulations of group delay times and phase dispersion spectra for two band plasmon induced transparency metamaterials are presented in Fig. 3. The shaded area shows the frequency interval in the limits of 70 % transmission. Fig. 3 shows that the frequency dependences of the phase shifts and group delay times on  $d$  are highly distinguishable for low- and high-frequency transparency windows. With the unit cell size  $d = 4.95$  mm, the group delay maximum is  $t_{gr1} = 3.63$  ns at  $f = 56.01$  GHz and decreases slowly with frequency in the low-frequency transparency window. In the high-frequency transparency window, the group delay is minimum  $t_{gr2} = 2.01$  ns at  $f = 56.14$  GHz, and it increases sharply up to  $t_{gr1} = 9.92$  ns at the high frequency  $f = 56.18$  GHz at the edge of the transparency band. At  $d = 5.1$  mm, the group delay maximum in the low-frequency transparency window is  $4.49$  ns at  $f = 55.44$  GHz, and it decreases quickly up to  $t_{gr1} = 2.21$  ns at  $f = 55.51$  GHz. In the high-frequency transparency window, the group delay is minimum  $t_{gr2} = 1.43$  ns at  $f = 55.61$  GHz, and it increases sharply up to  $t_{gr2} = 5.46$  ns at  $f = 55.68$  GHz, when the frequency increases. At  $d = 5.35$  mm, the group delay time maximum in the low-frequency transparency window is  $t_{gr1} = 7.46$  ns at  $f = 52.63$  GHz, and it decreases quickly to  $0.86$  ns at  $52.59$  GHz. In the high-frequency transparency windows, the group delay time is minimum  $t_{gr2} = 0.49$  ns at  $f = 53.04$  GHz and increases up to  $5.22$  ns at  $f = 53.15$  GHz. The results of the simulation of spectra for various  $d$  when two transparency windows exist are given in Table.

Data in the Table show that  $t_{gr1}$  for the low-frequency window increases almost linearly from  $1.17$  ns to  $7.46$  ns with increasing the lattice period from  $4.88$  mm to  $5.35$  mm. At the same time, the value  $t_{gr2}$  decreases sharply for the high-frequency transparency window from  $t_{gr2} = 17.41$  ns

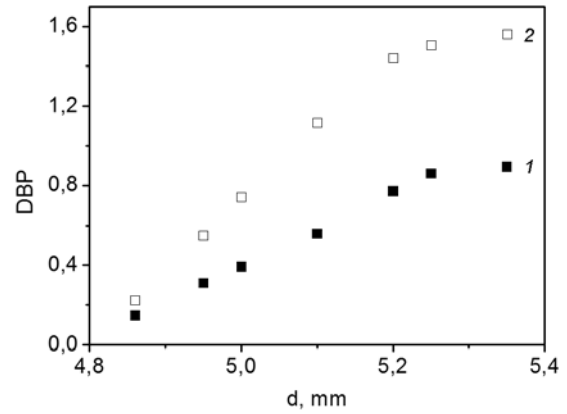


Fig. 4. Dependences of the delay-bandwidth-product parameter ( $DBP$ ) on  $d$  for low-frequency (1) and high-frequency (2) transparency windows.

at  $d = 4.88$  mm to  $4.87$  ns at  $d = 5.35$  mm. The group refractive index can be calculated from the information on the group delay time using the speed of light in vacuum  $c$  and the thickness of the substrate  $h_s$  as  $n_{gr1} = ct_{gr1}/h_s$ . Results in Table show very high values of group refractive indices for two transparency windows. These dependences mean that a microwave electromagnetic pulse with a central frequency located in the transparency windows will be significantly slowed down while propagating through the constructed unit cell with the triatomic metamolecules. Moreover, the group delay time and group refractive index can be controlled both by a unit cell dimension and the choice of a working frequency in the transparency windows. For a slow light device, the  $DBP$  parameter which is the product of the delay time and bandwidth is a measure of the quality (FOM-factor) [21].  $DBP$  is a measure of the capacitance of the delay line and also provides an estimate of the number of pulses that can be contained in a given medium [22]:

$$DBP = t_{gr1}\Delta f_{Ti}, \quad (3)$$

where  $\Delta f_{Ti}$  ( $i = 1, 2$ ) are the transparency bandwidth at the transparency coefficient  $0.7$  for low-frequency ( $i = 1$ ) or high-frequency ( $i = 2$ ) windows.  $DBP$  also provides an estimate on the number of pulses that can be transmitted via a communication channel [22]. Fig. 4 shows that the  $DBP$  parameter both for the low-frequency and high-frequency transparency windows increases almost linearly with lattice parameters, but the rate of change and the maxi-

mum  $DBP$  value are higher in the low-frequency transparency window.

The developed PIT metamaterial with two transparency windows has much larger  $DBP$  values than the highest  $DBP$  values reported in the literature in the microwave and terahertz ranges. In particular, the  $DBP = 0.39$  and the maximum  $t_{gr} = 1.92$  ns [23] and  $DBP = 0.45$  and  $t_{gr} = 0.27$  ns [21] for the frequency region near 6 GHz;  $DBP = 0.15$  and  $t_{gr} = 1.58$  ps [24] for the frequency region near 1 THz. The results clearly indicate a strong deceleration of the electromagnetic wave in the microwave range of the proposed meta-molecular configuration. The slow electromagnetic wave effect can be controlled over a wide range by the controlled changing of unit cell structure parameters.

#### 4. Conclusions

A computer model has been developed and a numerical simulation of a planar microwave metamaterial has been performed, which shows the effect of two-mode transparency in its transmission spectra. This effect is caused by the interaction of the "quasi-dark" surface lattice mode with the captured "dark" plasmon mode from a pair of "dark" meta-atoms. Correlations have been established between the parameters of the structure, i.e. the size of the unit cell and a pair of "dark" meta-atoms in the 3-meta-atomic unit cell of the metamaterial, as well as between the spectral transmittance and time delay of the excitation of an external electromagnetic wave and the group refractive index. It was found that the frequency dispersion of these parameters in the high-frequency and low-frequency transparency windows is different, which allows us to independently control the slowing down of electromagnetic wave parameters. The developed model of metamaterial group delay much higher values of deceleration parameters than those known so far.

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