

Electron gas magnetism in a semiconductor. Quantum well with a parabolic potential in a strong magnetic field

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Analytical expressions have been obtained for the density of states, reduced chemical potential, and magnetic susceptibility for a degenerate and nondegenerate electron gas in a quantum well in a strong magnetic field. It has been shown that as the magnetic field increases in the strongly degenerate case, the quantum levels below the Fermi level cross the Fermi level, resulting in jump oscillations of the density of states, reduced chemical potential, and magnetic susceptibility.

Keywords: reduced chemical potential, magnetic susceptibility, degenerate and non-degenerate electron gas, quantum well, confinement parabolic potential

Електронний газовий магнетизм у напівпровіднику. Квантова яма з параболічним потенціалом у сильному магнітному полі. Ханлар Гасанов, Джахангір Гусейнов, Фаміль Мамедов, Рагім Рагімов, Вусале Дадашова, Нагі Гасимов, Айнұра Гадієва, Ібрагім Аббасов

Отримано аналітичні вирази для щільності станів, наведеного хімічного потенціалу та магнітної сприйнятливості виродженого та невиродженого електронного газу в квантовій ямі у сильному магнітному полі. Показано, що при збільшенні магнітного поля в сильно виродженому випадку квантові рівні нижче рівня Фермі перетинають рівень Фермі, що призводить до стрибкоподібних коливань щільності станів, зменшення хімічного потенціалу та магнітної сприйнятливості.

1. Introduction

Nowadays, nanotechnology is one of the leading branches of science and technology, which has made revolutionary changes in industry. The study of the physical and chemical properties of low-dimensional electronic systems, the electronic properties of nanostructures associated with quantum size effects is developing very rapidly. Research on magnetic phenomena at the microscopic level is important for understanding the nature of quantum phenomena that mediate magnetic interactions in size-limited materials, as well as for developing cost-effective, miniature, and energy-efficient spintronic devices of a new generation [1]. The size quantization effect makes it possible to control the physical properties of quantum layers and create devices with the required character-

istics [2, 3]. The most important component of nanotechnologies are nanomaterials with unusual functional properties determined by the ordered structure of nanofragments in the size range from 1 to 100 nm. Achievements in this field lead to revolutionary developments in medicine, electronics, mechanical engineering and the creation of artificial intelligence.

Experimental and theoretical studies of the thermodynamic and galvanomagnetic properties of quantum dots, quantum wires, and quantized thin films are of great importance [2, 4-7]. From the calculation of the density of non-dissipative conducting currents in a quantum layer in an external magnetic field, a well-known universal expression has been obtained expressing the thermo-emf in a quantizing magnetic field in terms of the entropy of a unit volume of a gas of free charge carriers [8]. The diagonal coefficients of the dissipation tensor have been calculated, which make it possible to calculate the transverse galvanomagnetic and thermomagnetic effects based on the expression for the dissipative current density in a quantum wire [9]. In [10], the theory of thermo-emf of phonon drag for a one-dimensional electron gas in a quantum wire with a confinement parabolic potential has been developed, and in the work [11] - the theory of thermo-emf of drag arising in the presence of a temperature gradient in the plane of a layer of two-dimensional electron gas in a parabolic quantum well.

In [1], in quantum dots with a low capacitance, oscillations have been observed depending on the magnetic field for thermo-emf. An analogous phenomenon has been discovered for quantum wires in [4, 5, 12]. It has been shown in [13] that when the magnetic field changes, the electromagnetic eigenmodes of semiconductor nanotubes filled with a nonmetallic dielectric can theoretically experience Aharonov-Bohm oscillations. Therefore, this dependence on the magnetic field must be repeated for the thermodynamic parameters of the electron gas in the quantum well and wire.

The study of magnetic phenomena at the microscopic level has become an important area of research for low-dimensional magnetic materials. Understanding the quantum phenomena underlying magnetic interactions in size-limited materials is crucial for the development of a new generation of cheaper, compact and energy-efficient spintronics devices. In the work [14], carried out by V. Zawadzki on the magnetic oscillator of a two-dimensional electron gas, numerical calculations of the parameters of the GaAs sample have been provided, it has been shown that the diamagnetic moment fluctuates around zero depending on the magnetic field, and the paramagnetic and diamagnetic susceptibilities disappear in small areas for a two-dimensional electron gas. In [15], the main thermodynamic functions and the diamagnetic moment of the electron gas on the surface of a semiconductor nanotube were calculated at low temperature and in a weak magnetic field. Of special interest is the study of the dependence of the magnetic sensitivity on the magnetic field in semiconductor quantum wells with a parabolic potential.

The magnetic susceptibility for electron gas in large crystals was calculated by Yu.B. Rumer [16, 17]. In the given paper, the energy spectrum and wave functions have been calculated for the case when the magnetic field is parallel to the surface of the quantum well, as well as the dependence of the density of electron states, reduced chemical potential, and magnetic susceptibility on the magnetic field in a quantum well located in parallel magnetic field has been investigated. The confinement potential of the two-dimensional electron gas is selected to be parabolic.

2. Theory

2.1. Energy spectrum and wave function of electrons in a quantum well located in a longitudinal magnetic field

One of the pressing issues is the calculation of the energy spectrum and the wave function of the electron gas in a quantum well [18, 19]. The most frequently used in practice are quantum wells based on GaAs/Al_xGa_{1-x}As. Quantum wells with a parabolic potential can be obtained by alternating layers GaAs and Al_xGa_{1-x}As of different thicknesses in heterostructures. The relative thickness of the Al_xGa_{1-x}As layers should increase quadratically as they move away from the center of the well, and the thickness of the GaAs layers should decrease. Typically, layers with a thickness of about 1 nm are used. In a relatively wide quantum well ($L \gg 100$ nm), the potential can be created in the form of a parabola of depth Δ_1 , limited by a height barrier Δ_2 [19]. We will consider cases where the average electron energy $\bar{\epsilon}$ is less than the depth of the parabola (Δ_1). For the Fermi level, $\epsilon_f < \Delta_1$ for a degenerate electron gas, and $k_0T \ll \Delta_1$ for a non-degenerate

electron gas. Under these conditions, the electron confinement potential in the quantum well can be written as follows:

$$U(x) = \frac{1}{2} m \omega_0^2 x^2 \tag{1.1}$$

Where the x axis is directed perpendicular to the electron gas layer; m is the effective mass of an electron in the conduction band; ω_0 is the frequency characterizing the parabolic potential in the conduction band and is determined from the condition $\Delta_1 = \frac{m \omega_0^2 L_x^2}{8}$. Here L_x is the quantum well width.

The single-particle Schrödinger equation can be written as follows

$$\left[\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \omega_c p_y x + \frac{1}{2} m \omega^2 x^2 \right] \Psi = \epsilon \Psi \tag{1.2}$$

Here $\omega_c = \frac{eB}{mc}$ is the cyclotron frequency of the electron, $\omega = \sqrt{\omega_0^2 + \omega_c^2}$ is the “hybrid” frequency. Since the motion in the plane yz is free, we will seek the solution of Eq. (1.2) as follows

$$\psi = \varphi(x) e^{\frac{i}{\hbar}(p_y y + p_z z)} \tag{1.3}$$

Here $p_{y,z} = \hbar k_{y,z}$, $k_{y,z}$ are the components along the y and z axis of the electron wave vector.

Substituting expression (1.3) into (1.2), we can obtain the following expressions for the energy eigenvalue and wave eigenfunctions, considering the spin of the Schrödinger equation [20]

$$\epsilon = \left(\frac{1}{2} + N \right) \hbar \omega + \left(\frac{\omega_0}{\omega} \right)^2 \frac{\hbar^2 k_y^2}{2m} + \frac{\hbar^2 k_z^2}{2m} + \sigma g \mu_B B \tag{1.4}$$

$$\psi(x, y, z) = \frac{1}{\sqrt{L_y L_z}} \varphi_N(x - x_0) e^{i(p_y y + p_z z)} \tag{1.5}$$

Here: $\sigma = \pm \frac{1}{2}$ is the spin quantum number, g is the effective g – factor of an electron in a crystal, μ_B is the Bohr magneton, N is the oscillatory quantum number, L_y and L_z are the lengths of the system in the y and z directions.

$$\varphi_N(x - x_0) = \frac{1}{\sqrt[4]{\pi} \sqrt{R} \sqrt{2^N N!}} \exp \left[- \left(\frac{x - x_0}{\sqrt{2R}} \right)^2 \right] H_N \left(\frac{x - x_0}{R} \right) \tag{1.6}$$

$H_N(x)$ is the Hermite polynomial [21].

2.2. Density of states of a two-dimensional electron gas in a parallel magnetic field

Such features as the oscillation of kinetic coefficients and negative magnetoresistance are associated with the density of states of a two-dimensional electron gas in a quantizing magnetic field. The density of states of a two-dimensional electron gas is directly determined by the following general expression

$$g(\mathcal{E}) = \sum_{\alpha} \delta(\mathcal{E} - \mathcal{E}_0) \tag{2.1}$$

In expression (2.1), summation is carried out over all quantum numbers, including spin. $\delta(\epsilon)$ is the Dirac delta function. Using expression (1.4), if we pass from summation over k_y and k_z to integration, then integration over k_z is easily done using the delta function

$$g(\mathcal{E}) = \frac{L_y L_z}{(2\pi)^2} \frac{\sqrt{2m}}{\hbar} \sum_{N, \alpha} \int_{-\infty}^{\infty} \frac{dk_y}{\sqrt{\epsilon - \left(\frac{\omega_0}{\omega} \right)^2 \frac{\hbar^2 k_y^2}{2m} - \mathcal{E}_{N, \alpha}}} \tag{2.2}$$

Here

$$\mathcal{E}_{N,\alpha} = \left(\frac{1}{2} + N\right)\hbar\omega + \sigma g_{\mu_B} B \tag{2.3}$$

If in expression (2.2) we pass from integration over k_y to integration over the center of oscillations x_0 using the replacement

$$k_y = -\frac{m\omega}{\hbar} \frac{\omega}{\omega_c} x_0 \tag{2.4}$$

then for the density of states we obtain:

$$g(\mathcal{E}) = \frac{L_y L_z m}{2\pi\hbar} \frac{\omega}{\omega_0} \sum_{N,\alpha} I(N,\sigma,\mathcal{E}) \tag{2.5}$$

$$I(N,\sigma,\mathcal{E}) = \int_0^{x_{0m}} \frac{dx_0}{\sqrt{\frac{\mathcal{E} - \mathcal{E}_{N,\alpha}}{\beta} - x_0^2}} \tag{2.6}$$

$$\beta = \frac{m\omega_0^2 \omega^2}{2\omega_c^2} \tag{2.7}$$

In expression (2.6), the upper limit of the integral (x_{0m}), generally speaking, depends on N, σ, \mathcal{E} and L_x . To determine x_{0m} , we take into account that the summation in expressions (2.5)-(2.6) is carried out over Landau oscillators $\left(|x_0| < \frac{L_x}{2}\right)$, which radical expression is positive and centers are inside the quantum well.

Let's consider the following particular cases:

1) $\mathcal{E} - \mathcal{E}_{N,\sigma} < 0$. In this case $I(N,\sigma, \mathcal{E}) = 0$

2) $\mathcal{E} - \mathcal{E}_{N,\sigma} \leq \beta \left(\frac{L_x}{2}\right)^2$. In this case, the centers of all Landau oscillators corresponding to positive values of the radical expression are inside the quantum well, therefore, as can be seen from

$x_{0m} = \sqrt{\frac{\mathcal{E} - \mathcal{E}_{N,\sigma}}{\beta}}$ and (2.6), the integral $I(N,\sigma, \mathcal{E})$ does not depend on \mathcal{E} and L_x :

$$I(N,\sigma, \mathcal{E}) = \frac{\pi}{2} \tag{2.8}$$

3) $\mathcal{E} - \mathcal{E}_{N,\sigma} > \beta \left(\frac{L_x}{2}\right)^2$. In this case $x_{0m} = \frac{L_x}{2}$, and $I(N,\sigma, \mathcal{E})$ depends on \mathcal{E}, N , and L_x :

$$I(N,\sigma, \mathcal{E}) = \arcsin \frac{\sqrt{\beta} L_x}{2\sqrt{\mathcal{E} - \mathcal{E}_{N,\sigma}}} \tag{2.9}$$

Thus, if we consider expressions (2.8) and (2.9) in the expression for the density of states (2.5), then for the density of states we obtain the following expression:

$$g_H(\mathcal{E}) = \frac{L_y L_z m \omega}{2\pi\hbar^2 \omega_0} \left\{ \sum_{N,\sigma} H \left(-\mathcal{E} + \mathcal{E}_{N,\sigma} + \beta \left(\frac{L_x}{2}\right)^2 \right) H(\mathcal{E} + \mathcal{E}_{N,\sigma}) + \frac{2}{\pi} \sum_{N,\sigma} H \left(\mathcal{E} - \mathcal{E}_{N,\sigma} - \beta \left(\frac{L_x}{2}\right)^2 \right) \arcsin \frac{L_x \sqrt{\beta}}{2\sqrt{\mathcal{E} - \mathcal{E}_{N,\sigma}}} \right\} \tag{2.10}$$

Here $H(x)$ is the Heaviside function [22]:

$$H(x) = \begin{cases} 1, & \text{for } x \geq 0; \\ 0, & \text{for } x < 0. \end{cases}$$

It follows from the analysis of expression (2.10) that the density of states of an electron, when the magnetic field is parallel to the plane of the two-dimensional electron gas, does not consist of pure steps, as in the case of the absence of a magnetic field. Each time the electron energy is equal to the subband energy, the density of states changes abruptly with a finite step. Since the density of states at the previous level is non-zero, the density of states increases at each successive level.

In the majority of the cases studied in the experiment, the average electron energy ($\bar{\mathcal{E}}$) satisfies the condition

$$\bar{\mathcal{E}} - \frac{\hbar\omega}{2} - \frac{1}{2}|g|\mu_B B < \Delta_1 < \frac{\beta L_x^2}{2}$$

In this case, the second term in expression (2.10) for the density of electron states can be omitted, and $g(\mathcal{E})$ becomes a step function of \mathcal{E} :

$$g_H(\mathcal{E}) = \frac{L_x L_y}{2\pi\hbar^2} \frac{\omega}{\omega_0} \sum_{N,\sigma} H(\mathcal{E} - \mathcal{E}_{N,\sigma}) \tag{2.11}$$

Therefore, the density of states of a two-dimensional electron gas in a parallel magnetic field is a sawtooth function of the magnetic field at each given energy value.

2.3. Chemical potential of electrons in a quantum well

If we take the bottom of the quantum well as the energy reference point, then the number of electrons per unit surface in a two-dimensional electron gas is determined by the following expression:

$$n = \frac{1}{L_y L_z} \int_0^{\mathcal{E}} g(\mathcal{E}) f_0(\mathcal{E}) d\mathcal{E} \tag{3.1}$$

Here

$$f_0(\mathcal{E}) = \left[1 + \exp\left(\frac{\mathcal{E} - \xi}{k_0 T}\right) \right]^{-1} \tag{3.2}$$

is the distribution function of electrons in the state of equilibrium (Fermi-Dirac distribution), ξ is the chemical potential of electrons. If we substitute the expression for the density of states (2.11) into (3.1), then for the surface density of electrons we obtain

$$n = \frac{m}{2\pi\hbar^2} \frac{\omega}{\omega_0} \sum_{N,\sigma} \int_{\mathcal{E}_{N,\sigma}}^{\infty} \frac{d\mathcal{E}}{e^{\frac{\mathcal{E} - \xi}{k_0 T} + 1}} \tag{3.3}$$

After integrating the resulting expression over energy and summing over the spin quantum number, we obtain

$$n = \frac{n_0}{2} \frac{\omega}{\omega_0} \sum_N \left\{ \ln \left[e^{\eta - \left(N + \frac{1}{2}\right)a + \frac{b}{2}} + 1 \right] + \ln \left[e^{\eta - \left(N + \frac{1}{2}\right)a - \frac{b}{2}} + 1 \right] \right\} \tag{3.4}$$

Here it is marked:

$$n_0 = \frac{mk_0 T}{\pi\hbar^2}, \quad a = \frac{\hbar\omega}{k_0 T}, \quad b = \frac{|g|\mu_B B}{k_0 T}, \quad \eta = \frac{\xi}{k_0 T}. \tag{3.5}$$

The arbitrary degree of degeneracy of the two-dimensional electron gas and the chemical potential of electrons for an arbitrary magnetic field can be found from expression (3.4).

Below we examine special cases.

A. Nondegenerate electron gas. The condition of non-degeneracy of electrons in a quantum well ($f_0(\bar{\mathcal{E}}) \ll 1$) can be expressed as follows:

$$-\eta_0 \equiv \frac{a}{2} - \frac{b}{2} - \eta \gg 1 \tag{3.6}$$

Let us consider that in this case $\ln(1 + e^{\eta-x}) \approx e^{\eta-x}$ and in (3.4) we restrict ourselves to summation over N by the first term. This state is called the quantum limit state. Thus, for the chemical potential in the quantum limit state, we obtain:

$$\xi(B) = k_0 T \ln \left[\frac{2sh \frac{a}{2} n \omega_0}{2ch \frac{b}{2} n_0 \omega} \right] \tag{3.7}$$

From expressions (3.6) and (3.7) we can find the temperature range that satisfies the condition of a nondegenerate electron gas for a given value of the electron surface density. For example, for GaAs (when $m = 0,067$, $\hbar\omega = 2,91\text{meV}$ [23]), in order for the electron gas to become nondegenerate ($\eta_0 \leq -3$), the following conditions must be satisfied: at surface density $n = 10^8 \text{sm}^{-2}$, $T \geq 1\text{K}$, at surface density $n = 10^9 \text{sm}^{-2}$, $T \geq 8\text{K}$, and for $n = 10^{10} \text{sm}^{-2}$, $T \geq 60\text{K}$.

In the special case, if $a \gg 1, b \gg 1$, (3.7) takes the following form:

$$\xi(B) = \frac{\hbar\omega}{2} - \frac{1}{2} |g| \mu_B B + k_0 T \ln \left(\frac{2n \omega_0}{n_0 \omega} \right) \tag{3.8}$$

And for the condition $a \gg 1, b \ll 1$ for the chemical potential in a non-degenerate form, we obtain

$$\xi(B) = \frac{\hbar\omega}{2} + k_0 T \ln \left(\frac{2n \omega_0}{n_0 \omega} \right) \tag{3.9}$$

B. Strongly degenerate electron gas. Condition (3.6) is not satisfied for a degenerate electron gas. Then for a strongly degenerate electron gas:

$$-\eta_0 \equiv -\frac{a}{2} + \frac{b}{2} \gg 1 \tag{3.10}$$

When this condition is met, one can be discarded in the argument of the logarithmic functions in expression (3.4) (which is equivalent to replacing the function $\left[-\frac{\partial f_0}{\partial \mathcal{E}}\right]$ with $\delta(\mathcal{E} - \xi)$), and in this case, the chemical potential in any magnetic field can be calculated using the resulting expression. The analytical dependence of the chemical potential of the degenerate state on the magnetic field depends on the region in which the energy of the electron is located. If the chemical potential located between spin-split sublevels of the same Landau level, i.e. if $\left(N + \frac{1}{2}\right)a - \frac{b}{2} < \eta < \left(N + \frac{1}{2}\right)a + \frac{b}{2}$, then from expression (3.4) for the chemical potential we obtain:

$$\xi(B) = \left[\frac{(N_0 - 1)^2}{4L} + \frac{1}{2} \right] \hbar\omega + \frac{2k_0 T}{L} \frac{n \omega_0}{n_0 \omega} - \frac{|g| \mu_B B}{2L} \tag{3.11}$$

Here $N_0 = \frac{L}{2} - 1$ is the number of the largest of the quantized levels of the oscillator below the chemical potential. When the chemical potential is between adjacent Landau levels, i.e. when

$$\left(N + \frac{1}{2}\right)a + \frac{b}{2} < \eta < \left(N + \frac{3}{2}\right)a, \quad \xi(B) = \frac{L}{4}\hbar\omega + \frac{2k_0T}{L} \frac{n}{n_0} \frac{\omega}{\omega_0} \quad (3.12)$$

Here L represents the total number of quantized sublevels below the chemical potential. For a given value of the surface density, the dependence of the energy of the N th Landau level on the magnetic

field has the form $\left(N + \frac{1}{2}\right)\hbar\omega_0 \left(1 + \frac{\omega_c^2}{\omega_0^2}\right)^{\frac{1}{2}}$. And $\xi(B)$ weakly depends on the magnetic field, since, as

can be seen from expressions (3.11) and (3.12), as the magnetic field increases, the first terms increase, and the second terms decrease, so the changes partially compensate each other. Therefore, as the magnetic field increases, the quantized levels of the oscillator alternately cross the chemical potential and move from bottom to top. In this case, expressions (3.11) and (3.12) are alternately

used for $\xi(B)$. It is clear that for transitional values of $\eta = \left(N_0 + \frac{1}{2}\right)a - \frac{b}{2}$ and $\eta = \left(N_0 + \frac{1}{2}\right)a + \frac{b}{2}$, expressions (3.11) and (3.12) must coincide. (3.11) and (3.12) satisfy this condition, but it should be considered that when passing from (3.11) to (3.12) or vice versa (for transitional energies), L must change by one unit. If we neglect the spin splitting ($b \rightarrow 0$), then we can write a general expression for the chemical potential in the entire region of the magnetic field:

$$\xi(B) = (N_0 + 1)\frac{\hbar\omega}{2} + \frac{k_0T}{N_0 + 1} \frac{n}{n_0} \frac{\omega}{\omega_0} . \quad (3.13)$$

Since the value of g in **GaAs** is small, the results obtained from expression (3.13) almost coincide with the results obtained from expressions (3.11) and (3.12). N_0 is found from the following condition

$$\left(N_0 + \frac{3}{2}\right)\hbar\omega > \xi(B) \geq \left(N_0 + \frac{1}{2}\right)\hbar\omega \quad (3.14)$$

and through the chemical potential depends on the concentration and the magnetic field. Thus, using expressions (3.13) and (3.14), for N_0 we obtain the following expression

$$N_0 = \left\lfloor \sqrt{\frac{2\pi\hbar\omega_0}{m\omega^2} + \frac{1}{4}} - \frac{1}{2} \right\rfloor \quad (3.15)$$

Here the integer part of the number x is denoted by $[x]$.

For a given value of surface density, the value of the magnetic field at which the chemical potential crosses any (N th) level is found from expression (3.13)

$$B_N = \frac{m\omega_0}{e} \sqrt{\frac{2\pi n\hbar}{m\omega_0 N(N-1)}} - 1 . \quad (3.16)$$

It should be noted that in the strongly degenerate case, the chemical potential is above the 0th Landau level for all values of the magnetic field, i.e. in (3.16) $N \geq 1$. Even in the absence of a magnetic field, at surface densities $n < \frac{m\omega_0}{\pi\hbar}$, the chemical potential remains below the 1st Landau level, and the electrons are mostly at the 0th Landau level (quantum limit state). In this case, the chemical potential

$$\xi(B) = \frac{\hbar\omega}{2} + \frac{\pi\hbar^2}{m} n \frac{\omega}{\omega_0} \quad (3.17)$$

2.4. Magnetic susceptibility in a quantum well of an electron gas in a quantizing magnetic field

Let us calculate the magnetic susceptibility of an electron gas in a quantum well with a parabolic potential located in a longitudinal magnetic field for the statistics of nondegenerate and degenerate states.

A. Non-degenerate state. We will use a grand potential to calculate the magnetic susceptibility. It is known that the Gibbs thermodynamic potential

$$\Omega = -k_0T \sum_{N,k_y,k_z,\sigma} \ln \left(1 + \exp \left(\frac{\xi - \mathcal{E}_{N,k_y,k_z,\sigma}}{k_0T} \right) \right) \quad (4.1)$$

Magnetic susceptibility [16]

$$\chi(B,T) = \frac{M}{B} = - \frac{1}{VB} \left(\frac{\partial \Omega}{\partial T} \right)_{\xi,T} \quad (4.2)$$

It is obviously that for the statistics of a nondegenerate state

$$\Omega = -nk_0T \quad (4.3)$$

Here n is the surface density of electrons. Substituting expression (1.7) into (4.1) and proceeding to integration, from expression (4.2) we obtain the following expression

$$\chi = \frac{1}{L_x B^2} nk_0T \frac{\omega_c^2}{\omega^2} (1 + \nu_s) \frac{\omega^2}{\omega_c^2} th\nu_s - \nu cth\nu - \frac{t \exp(-t^2)}{\sqrt{\pi} \operatorname{erf}(t) \frac{\omega_c^2}{\omega_0^2}} \quad (4.4)$$

Here $t = \sqrt{\frac{bL_x^2}{4k_0T}}$, $\operatorname{erf}(t)$, is the integral of probabilities [24]. In addition, in expression (4.4) the following notation is used:

$$\nu = \frac{\hbar\omega}{2k_0T}, \nu_s = \frac{g\mu_B B}{2k_0T}, \eta = \frac{\xi}{k_0T} \quad (4.5)$$

Particularly, using the work [25] for InSb and GaAs, the dependences $\chi(B)$ have been established. Here, assuming the well width $L_x = 4000\text{\AA}$, the height of the parabolic well $\Delta_1 = 150\text{ meV}$, the electron mass $m = 0.067m_0$, then for **GaAs** we obtain $\omega_0 = 4.437 \cdot 10^{12} \text{ s}^{-1}$. It should be noted that in the absence of a magnetic field $\Delta_1 \equiv \frac{m\omega_0^2}{2} \left(\frac{L_x}{2} \right)^2$ [23]. For an InSb crystal, respectively, $m = 0.016m_0$, $\hbar\omega_0 = 7,5\text{ meV}$. In addition, for **InSb** and **GaAs**, respectively, the concentration and Lande factor were chosen as:

$$\begin{aligned} n &= 2 \cdot 10^{10} \text{ cm}^{-2}; g = 51,2; \\ n &= 5 \cdot 10^{10} \text{ cm}^{-2}; g = -0,44. \end{aligned}$$

Figure 1 shows the dependence of $\chi(B)$ on the magnetic field for **InSb** (curve 1) and **GaAs**(curve 2), respectively.

B. Degenerate state. For a typical $\text{Al}_x\text{Ga}_{1-x}\text{As}$ [25], $x_0^m x_0^m$ is the degree of filling of the well according to the upper limit of the integral; in the case of $\mathcal{E}_F - \mathcal{E}_{N,\sigma} < b \left(\frac{L_x}{2} \right)^2$, the result of integration does not depend on the width of the well and is $\frac{\pi}{\sqrt{b}}$. Otherwise, that is, in the case of $\mathcal{E}_F - \mathcal{E}_{N,\sigma} > b \left(\frac{L_x}{2} \right)^2$, the integration result depends on the well width L_x . Since we are considering

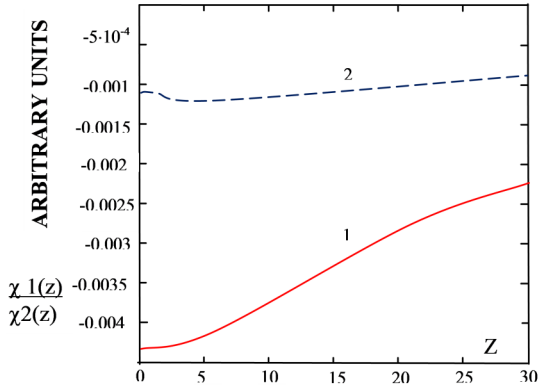


Fig. 1 Dependence of the magnetic susceptibility $\chi(B)$ on the magnetic field of InSb (curve 1) and GaAs (curve 2)

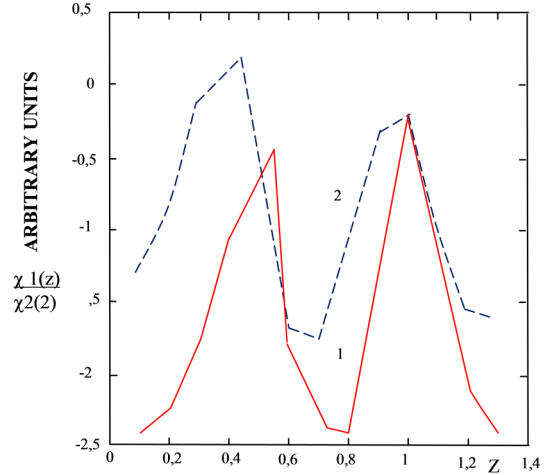


Fig. 2 Dependence of the magnetic susceptibility on the magnetic field of InSb (curve 1) and GaAs (curve 2)

the degenerate case, at given concentrations the Fermi level in the following formulas is much lower than $b\left(\frac{L_x}{2}\right)^2$. Therefore, the magnetic susceptibility in the degenerate state has an oscillatory character [26], changes nonmonotonically, and is written by the following formula:

$$\chi = \frac{e^2}{2\pi L_x m c^2} \cdot \frac{1}{2\sqrt{1+z^2}} \sum_{N,\sigma} \left(\eta - \sigma g \frac{m}{2m_0} z - \left(N + \frac{1}{2} \right) \sqrt{1+z^2} \right) \times \left(\eta - \sigma g \frac{m}{2m_0} \cdot \frac{(2+3z)}{z} - 3 \left(N + \frac{1}{2} \right) \sqrt{1+z^2} \right) \times H \left(\eta - \sigma g \frac{m}{2m_0} z - \left(N + \frac{1}{2} \right) \sqrt{1+z^2} \right) \quad (4.6)$$

It is indicated $z = \frac{\omega_c}{\omega_0}$, $\eta = \frac{\mathcal{E}_F}{\hbar\omega_0}$. In a magnetic field, the formula for the concentration n , expressed in terms of the Fermi level, is as follows.

$$n = \frac{m\omega_0}{2\pi\hbar} \sqrt{1+z^2} \sum_{N,\sigma} \left(\eta - \sigma g \frac{m}{2m_0} z - \left(N + \frac{1}{2} \right) \sqrt{1+z^2} \right) \times H \left[\eta - \sigma g \frac{m}{2m_0} - \left(N + \frac{1}{2} \right) \sqrt{1+z^2} \right] \quad (4.7)$$

Figure 2 shows the oscillatory dependence of the magnetic susceptibility (χ) on the magnetic field in InSb ($n = 10^{12} \text{ cm}^{-2}$) (curve 1) and GaAs ($n = 5 \cdot 10^{11} \text{ cm}^{-2}$) (curve 2) at $T=4,2 \text{ K}$. In contrast to the non-degenerate state, the concentration has been increased in order to feel several oscillations more clearly. Since the spectroscopic decay constant g for the InSb crystal is large, the jumps obtain a more nonsmooth character.

3. Conclusion

In order to substantiate the theory of thermodynamics and thermomagnetic kinetic phenomena of an electron gas in a parallel magnetic field in quantum wells with a parabolic confinement potential, the energy spectrum and wave function of electrons have been determined, the function of the density of states of a two-dimensional electron gas located in a parallel magnetic field has been found, and it has been shown that it is a sawtooth function of the magnetic field, the magnetic susceptibility of the electron gas has been calculated for the statistics of nondegenerate and degenerate states. The dependence of the magnetic susceptibility on the magnetic field in the degenerate state has an oscillatory character, and its absolute values change abruptly. It has been shown that the crossing of the Fermi level by quantum levels located below the Fermi level with increasing mag-

netic field strength causes jump oscillations of the density of states, reduced chemical potential, and magnetic susceptibility.

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